UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Department of Bioengineering

Homework Problems for Chapter 1.

1. For the following, draw each pair of vectors on a Cartesian axis. Then find their magnitudes and directions.

   a.
   $$\mathbf{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

   b.
   $$\mathbf{u} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \mathbf{w} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \qquad \mathbf{x} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

   Based on the plot of vectors in (b), which vectors are orthogonal?

2. Find unit vectors for the vectors of (1a).

3. For the vector pairs in (1a), find $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$. Also draw these vectors and find their magnitude and direction.

4. Find the inner products between all possible pairs of the following vectors.

   a.
   $$\mathbf{u} = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} -5 \\ -3 \\ -1 \\ 0 \end{pmatrix} \qquad \mathbf{w} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} \qquad \mathbf{x} = \begin{pmatrix} 8 \\ -8 \\ 2 \\ -2 \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} -8 \\ -8 \\ -2 \\ -2 \end{pmatrix}$$

   b.
   $$\mathbf{u} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \qquad \mathbf{w} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \qquad \mathbf{x} = \begin{pmatrix} 2 \\ 9 \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

5. Among the vectors in (4a), which are orthogonal? Among orthogonal vectors, how would you change them so they become orthonormal?

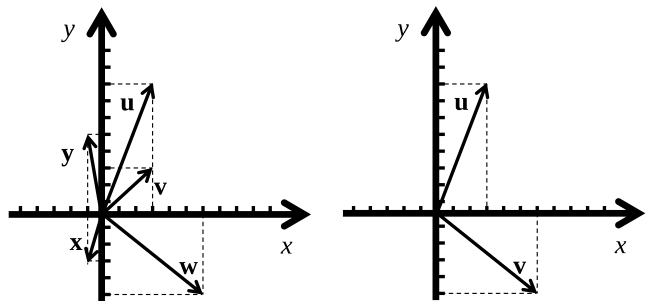

Figure 1: Figures related to Problem 6 (left) and Problem 7 (right).

6. Find the vector magnitudes and the angles between adjacent vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}, \mathbf{y}$ in the figure.

7. Find the projections of $\mathbf{u}$ onto the x and y axes from vectors in Fig. 1 (right). Do the same for $\mathbf{v}$.

8. Compute

$$\alpha \begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -5 \\ 2 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -1 \\ 3 \\ -3 \end{pmatrix}$$

For

a.

$$\alpha = 1, \ \beta = 2, \ \lambda = 1$$

b.

$$\alpha = -1, \ \beta = 2, \ \lambda = -1$$

9. Solve the following equation for x or explain why a solution does not exist.

$$2 \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + 3 \begin{pmatrix} x \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 18 \end{pmatrix}$$

10. Solve for α or explain why a solution does not exist.

$$\alpha \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \\ -8 \end{pmatrix}$$

11. (a) Find values of α, β, and λ for the following vector equation.

$$\alpha \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 21 \end{pmatrix}$$

(b) Solve for x_1, x_2, and x_3. Then please look ahead in the book to find a method using the augmented matrix and row operations that results in the same solution.

$$x_1 + x_2 + x_3 = 2$$

$$2x_1 + 3x_2 + x_3 = 3$$

$$x_1 - x_2 - 2x_3 = -6$$

12. Consider the vectors

$$\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad \mathbf{w} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

Express $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{w}$ (i.e., $\mathbf{w} = \alpha\mathbf{u} + \beta\mathbf{v}$)

13. For the basis vectors

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \mathbf{e_2} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

and the vectors

$$\mathbf{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

a. Verify the basis vectors are orthogonal.

b. Verify that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

c. Express $\mathbf{u}$ and $\mathbf{v}$ as linear combinations of the basis vectors (i.e., $\mathbf{u} = \alpha\mathbf{e_1} + \beta\mathbf{e_2}$ and $\mathbf{v} = \lambda\mathbf{e_1} + \gamma\mathbf{e_2}$)

d. Verify that the expanded versions of $\mathbf{u}$ and $\mathbf{v}$ are orthogonal by computing $(\alpha\mathbf{e_1} + \beta\mathbf{e_2})^T(\lambda\mathbf{e_1} + \gamma\mathbf{e_2})$

14. Prove the Cauchy-Schwarz Inequality: $||\mathbf{u}^T\mathbf{v}|| \leq ||\mathbf{u}||\ ||\mathbf{v}||$

15. Prove the Triangle Inequality: $||\mathbf{u} + \mathbf{v}|| \leq ||\mathbf{u}|| + ||\mathbf{v}||$